After integration we obtain:

$$S_e = \frac{8}{\pi^2} ab \approx 0.81 S .$$

Thus, due to the cosine amplitude distribution the area utilization factor is 0,81. In tab. 6.1 values of the area utilization factor as well for other distributions are given.

6.5. Influence of the phase distribution on the radiation field

The cophased distribution of tangential components of the field intensity in the aperture is uneasy to create, as owing to some design features or discrepancy of the aerial manufacturing, there are phase distortions in the aperture. Besides, in many cases in the aperture special laws of the phase distribution are used to form the special DD form or to scan the beam in space. Therefore skills for analyzing influence of the phase distribution on the radiation field are of great importance.

Generally, it is necessary to consider the non-uniform amplitude-phase distribution, at which both the amplitude and the phase of tangential components of intensity are the function of two coordinates in the aperture. For the sake of simplicity let us consider, that the intensity amplitude in all aperture points is identical, whereas the phase of the field intensity depends only on one coordinate, for instance, coordinate x. Then the field intensity in the aperture is

$$\dot{E}_S = E_m e^{-i\psi(x)}, \qquad (6.27)$$

where $\psi(x)$ is the dependence of the phase on the aperture coordinate x (distribution of the phase).

Let us spread out the function of the phase distribution in a power series on the dimensionless argument $2x/a$:

$$\psi(x) = \psi_1 \left(\frac{2x}{a} \right) + \psi_2 \left(\frac{2x}{a} \right)^2 + \psi_3 \left(\frac{2x}{a} \right)^3 + \ldots, \qquad (6.28)$$

where $\psi_1, \psi_2, \psi_3, \ldots$ are constant factors, which determine phase shift at the aperture edge (at $x = a/2$).

At the linear dependence of the phase on coordinate all factors of series (6.28), except ψ_1, are equal to zero. In this case expression (6.27) is reduced to the formula

$$\dot{E}_S = E_m e^{-i\psi_1 \frac{2x}{a}}. \qquad (6.29)$$

Substituting values of the field intensity (6.29) in formula (6.8) and considering, that $W = W_a$, $-a/2 \le x \le a/2$ and $-b/2 \le y \le b/2$, DC in the main planes can be found. For example, in plane XOZ:

$$f(\theta) = (1 + cos\,\theta)\frac{sin\left[\frac{ka}{2}\left(sin\,\theta - \frac{2\psi_1}{ka}\right)\right]}{\frac{ka}{2}\left(sin\,\theta - \frac{2\psi_1}{ka}\right)}. \qquad (6.30)$$

The above expression is similar to formula (5.26). At the linear phase change the direction of the maximal radiation displaces from normal to the aperture. The deviation level may be found from condition of equality to zero of argument in square brackets of the second factor in the right part of formula (6.30):

$$sin\,\theta_m = \frac{2\psi_1}{ka}.$$

The beamwidth slightly increases in comparison with the cophased excitation.

At the square-law phase distribution in series (6.28) only factor ψ_2 differs from zero, therefore, the field intensity in the aperture is determined as

$$\dot{E}_S = E_m e^{-i\psi_2 \left(\frac{2x}{a}\right)^2}. \qquad (6.31)$$

Substituting values (6.31) in formula (6.8) with $\varphi = 0$

$$\dot{E} = i\frac{E_m}{2\lambda r}(1 + cos\,\theta)e^{-ikr}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} e^{i\left(kx\,sin\,\theta - 4\psi_2\frac{x^2}{a^2}\right)}dxdy, \qquad (6.32)$$

where expression (3.23) is used as DC of the Huygens element.

The function under integral of expression (6.32) does not depend on y, therefore, having integrated over y and having substituted limits, we obtain factor b . The remained one-dimensional integral can be presented by multiplying into some constant $exp\left(-i0.5\pi B^2\right)$, thus:

$$\int_{-a/2}^{a/2} e^{-i\left(\frac{\pi}{2}B^2 - kx\sin\theta + 4\psi_2\frac{x^2}{a^2}\right)}dx = \int_{\upsilon_1}^{\upsilon_2} e^{-i\frac{\pi t^2}{2}}dt. \quad (6.33)$$

It enables us to express this correlation through the Fresnel integral, as

$$\int_0^{\upsilon} e^{-i\frac{\pi t^2}{2}}dt = C(\upsilon) - iS(\upsilon),$$

where $C(\upsilon)$ and $S(\upsilon)$ are Fresnel integrals:

$$C(\upsilon) = \int_0^{v} cos\left(\frac{\pi t^2}{2}\right)dt;$$

$$S(\upsilon) = \int_0^{v} sin\left(\frac{\pi t^2}{2}\right)dt.$$

Let us equate an exhibitor of exponents from equation (6.33) to value t^2

$$B^2 - \frac{2k}{\pi}x\sin\theta + \frac{8\psi_2}{\pi}\frac{x^2}{a^2} = t^2. \quad (6.34)$$

Hence constant B is determined as a value, which transforms the left part of equation (6.34) into a full square of difference of two members:

$$B = \frac{\sqrt{2}ka\sin\theta}{4\sqrt{\pi\psi_2}} = \frac{u}{\sqrt{2\pi\psi_2}}, \quad (6.35)$$

where $u = \frac{ka}{2}\sin\theta$.

Substituting expression (6.35) in equation (6.34), we can find values of variable t :

$$t = \frac{2\sqrt{2}}{a}\sqrt{\frac{\psi_2}{\pi}}x - \frac{u}{\sqrt{2\pi\psi_2}}.$$

As a result of replacement of a variable, expression (6.32) is written down as

$$\dot{E} = i \frac{E_m ab\sqrt{\pi}}{4\sqrt{2\psi_2}\lambda r}(1 + \cos\theta)e^{-i\left(kr - \frac{1}{2}\pi B^2\right)}\{[C(\upsilon_1) + C(\upsilon_2)] - i[S(\upsilon_1) + S(\upsilon_2)]\}$$

(6.36)

where

$$\upsilon_1 = \sqrt{\frac{2\psi_2}{\pi}} - \frac{u}{\sqrt{2\pi\psi_2}};$$

$$\upsilon_2 = \sqrt{\frac{2\psi_2}{\pi}} + \frac{u}{\sqrt{2\pi\psi_2}}.$$

From expression for the field intensity the array factor is determined as

$$f_\Sigma(\upsilon) = \sqrt{[C(\upsilon_1) + C(\upsilon_2)]^2 + [S(\upsilon_1) + S(\upsilon_2)]^2}. \qquad (6.37)$$

Calculated by formula (6.37) DDs of the rectangular aperture with the square-law phase distribution are represented in Fig. 6.5, where on an abscise axis the values of the generalized angular argument u are plotted. For comparison in Fig. 6*.5(a)* DD with the uniform phase distribution is shown. In Fig. 6*.5(b)* the maximal phase shift at the edge of the aperture would reach $\psi_2 = \pi/2$; in Fig. 6*.5(c)* shift $\psi_2 = \pi$ and in Fig. 6*.5(d)* - shift $\psi_2 = 1.5\pi$.

Submitted DDs show that: at the phase distribution with the square-law dependence the directions of the zero radiation in DD disappear and the radiation minimums appear instead. Besides, the major lobe extends and the level of side lobes grows. At significant phase shifts the major lobe forks.

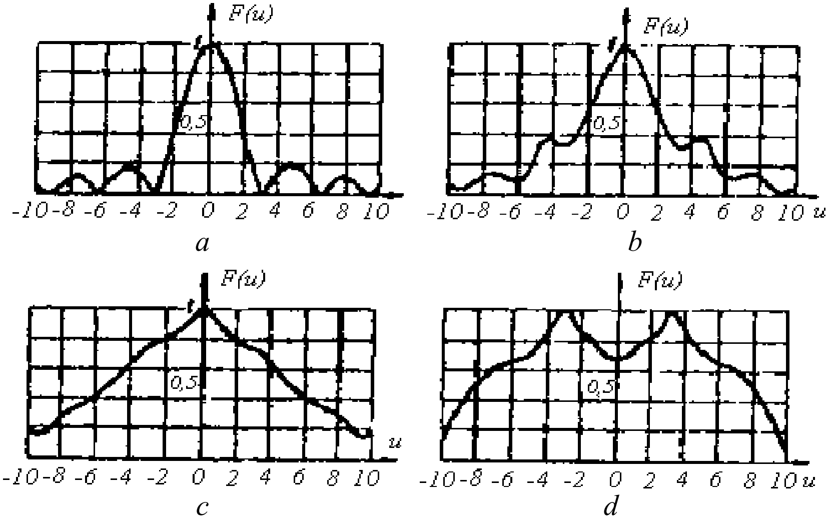


Fig. 6.5

In the case of the cosine amplitude distribution (6.24), and the phase, which is by under the square law (6.31) employing formula (6.8) we obtain:

$$\dot{E} = i\frac{E_m ab\sqrt{\pi}}{4\sqrt{2\psi_2}\lambda r}(1+\cos\theta)e^{-ikr}\Big\{e^{i\Phi_1}\big[C(\upsilon_1)+C(\upsilon_2)+C(\upsilon_3)\big]+$$

$$+e^{i\Phi_2}\big[C(w_1)+C(w_2)\big]-i\big[S(w_1)+S(w_2)\big]\Big\},$$

where

$$\Phi_1 = \frac{1}{8\psi_2}\left(\frac{\pi}{2}+u\right)^2;$$

$$\Phi_2 = \frac{1}{8\psi_2}\left(\frac{\pi}{2}-u\right)^2;$$

$$\upsilon_1 = \sqrt{\frac{2\psi_2}{\pi}}-\frac{1}{\sqrt{2\pi\psi_2}}\left(\frac{\pi}{2}+u\right);$$

$$\upsilon_2 = \sqrt{\frac{2\psi_2}{\pi}}+\frac{1}{\sqrt{2\pi\psi_2}}\left(\frac{\pi}{2}+u\right);$$

$$w_1 = \sqrt{\frac{2\psi_2}{\pi}} + \frac{1}{\sqrt{2\pi\psi_2}}\left(\frac{\pi}{2} - u\right);$$

$$w_2 = \sqrt{\frac{2\psi_2}{\pi}} - \frac{1}{\sqrt{2\pi\psi_2}}\left(\frac{\pi}{2} - u\right).$$

The module of the expression in braces is the array factor, which at big values a defines the type of DC. So in Fig. 6.6*(a)* DDs for the cophased and cosine amplitude distribution of the electric field intensity E are presented, in Fig. 6.6*(b)* - DD at $\psi_2 = \pi/2$, in Fig. 6.6(*c*) - DD at $\psi_2 = \pi$ and in Fig. 6.6*(d)* - DD at $\psi_2 = 1.5\pi$. Comparing these DDs, and that represented in Fig. 6.5, we can make a conclusion, that if the amplitude distribution falls down to aperture edges, the influence of the phase distribution of the directional

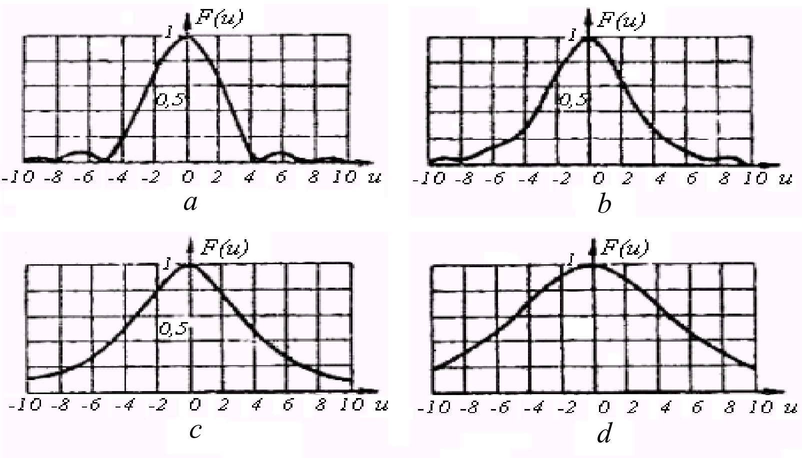
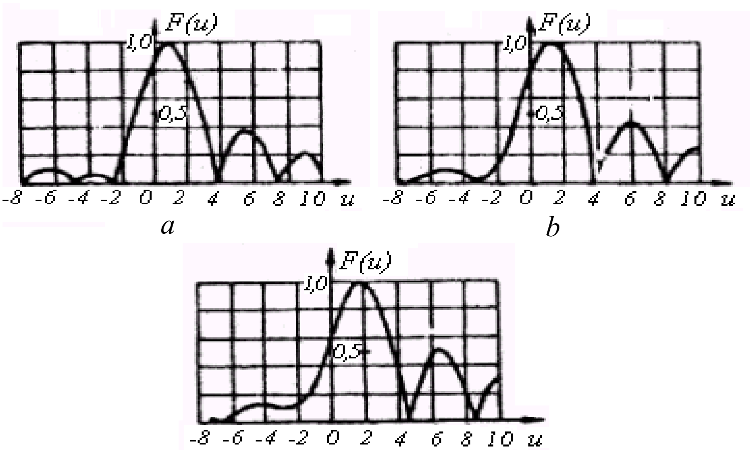


Fig. 6.6

properties of the aerial decreases. This is due to the fact that at the nonuniform distribution simultaneously with the growth of phase shifts the amplitude of the field intensity falls. Therefore the radiation of aperture elements, which are excited with a significant phase shift, will be essential smaller than the radiations of the aperture elements with small phase shifts.

In the case of the cubic distribution of the phase in the aperture DD becomes asymmetrical (Fig. 6.7*(a)* at $\psi_3 = \pi/2$, Fig. 6.7*(b)* at $\psi_3 = 0.75\pi$, Fig. 6.7*(c)* at $\psi_3 = \pi$). The direction of the maximal radiation deviates from a normal to the aperture aside lagging of a phase. Side lobes are increased from the side in which the direction of the maximal radiation deviates. On the other hand side lobes decrease.



c

Fig. 6.7

Submitted DDs illustrate, that phase distortions affect the aperture properties. The strongest influence of phase distortions takes place at the uniform amplitude excitation.